

K-8 Mathematics Grade-Level Indicators

**Number, Number Sense and
Operations Standard**

Kindergarten

*Number and
Number
Systems*

1. Compare and order whole numbers up to 10.
2. Explain rules of counting, such as each object should be counted once and that order does not change the number.
3. Count to twenty; e.g., in play situations or while reading number books.
4. Determine “how many” in sets (groups) of 10 or fewer objects.
5. Relate, read and write numerals for single-digit numbers (0 to 9).
6. Construct multiple sets of objects each containing the same number of objects.
7. Compare the number of objects in two or more sets when one set has one or two more, or one or two fewer objects.
8. Represent and use whole numbers in flexible ways, including relating, composing and decomposing numbers; e.g., 5 marbles can be 2 red and 3 green or 1 red and 4 green.
9. Identify and state the value of a penny, nickel and dime.

*Meaning of
Operations*

10. Model and represent addition as combining sets and counting on, and subtraction as take-away and comparison. For example:
 - a. Combine and separate small sets of objects in contextual situations; e.g., add or subtract one, two, or another small amount.
 - b. Count on (forward) and count back (backward) on a number line between 0 and 10.
11. Demonstrate joining multiple groups of objects, each containing the same number of objects; e.g., combining 3 bags of candy, each containing 2 pieces.
12. Partition or share a small set of objects into groups of equal size; e.g., sharing 6 stickers equally among 3 children.

*Computation
and Estimation*

13. Recognize the number or quantity of sets up to 5 without counting; e.g., recognize without counting the dot arrangement on a domino as 5.

Grade One

*Number and
Number Systems*

1. Use ordinal numbers to order objects; e.g., first, second, third.
2. Recognize and generate equivalent forms for the same number using physical models, words and number expressions; e.g., concept of ten is described by “10 blocks”, full tens frame, numeral 10, $5 + 5$, $15 - 5$, one less than 11, my brother’s age.
3. Read and write the numerals for numbers to 100.
4. Count forward to 100, count backwards from 100, and count forward or backward starting at any number between 1 and 100.
5. Use place value concepts to represent whole numbers using numerals, words, expanded notation and physical models with ones and tens. For example:
 - a. Develop a system to group and count by twos, fives and tens.
 - b. Identify patterns and groupings in a 100's chart and relate to place value concepts.
 - c. Recognize the first digit of a two-digit number as the most important to indicate size of a number and the nearness to 10 or 100.
6. Identify and state the value of a penny, nickel, dime, quarter and dollar.
7. Determine the value of a small collection of coins (with a total value up to one dollar) using 1 or 2 different type coins, including pennies, nickels, dimes and quarters.
8. Show different combinations of coins that have the same value.
9. Represent commonly used fractions using words and physical models for halves, thirds and fourths, recognizing fractions are represented by equal size parts of a whole and of a set of objects.

*Meaning of
Operations*

10. Model, represent and explain addition as combining sets (part + part = whole) and counting on. For example:
 - a. Model and explain addition using physical materials in contextual Situations.
 - b. Draw pictures to model addition.
 - c. Write number sentences to represent addition.
 - d. Explain that adding two whole numbers yields a larger whole number.
11. Model, represent and explain subtraction as take-away and comparison. For example:
 - a. Model and explain subtraction using physical materials in contextual situations.
 - b. Draw pictures to model subtraction.
 - c. Write number sentences to represent subtraction.
 - d. Explain that subtraction of whole numbers yields an answer smaller than
the original number

Grade Two

*Number and
Number Systems*

1. Use place value concepts to represent, compare and order whole numbers using physical models, numerals and words, with ones, tens and hundreds. For example:
 - a. Recognize 10 can mean “10 ones” or a single entity (1 ten) through physical models and trading games.
 - b. Read and write 3-digit numerals (e.g., 243 as two hundred forty three, 24 tens and 3 ones, or 2 hundreds and 43 ones, etc.) and construct models to represent each.
2. Recognize and classify numbers as even or odd.
3. Count money and make change using coins and a dollar bill.
4. Represent and write the value of money using the ¢ sign and in decimal form when using the \$ sign.
5. Represent fractions (halves, thirds, fourths, sixths and eighths), using words, numerals and physical models. For example:
 - a. Recognize that a fractional part can mean different amounts depending on the original quantity.
 - b. Recognize that a fractional part of a rectangle does not have to be shaded with contiguous parts.
 - c. Identify and illustrate parts of a whole and parts of sets of objects.
 - d. Compare and order physical models of halves, thirds and fourths in relations to 0 and 1.

*Meaning of
Operations*

6. Model, represent and explain subtraction as comparison, take-away and part-to-whole; e.g., solve missing addend problems by counting up or subtracting, such as “I had six baseball cards, my sister gave me more, and I now have ten. How many did she give me?” can be represented as $6 + ? = 10$ or $10 - 6 = ?$.
7. Model, represent and explain multiplication as repeated addition, rectangular arrays and skip counting.
8. Model, represent and explain division as sharing equally and repeated subtraction.
9. Model and use the commutative property for addition.

*Computation
and Estimation*

10. Demonstrate fluency in addition facts with addends through 9 and corresponding subtractions; e.g., $9 + 9 = 18$, $18 - 9 = 9$.
11. Add and subtract multiples of 10.
12. Demonstrate multiple strategies for adding and subtracting 2- or 3-digit whole numbers, such as:
 - a. compatible numbers;
 - b. compensatory numbers;
 - c. informal use of commutative and associative properties of addition.

Grade Three

*Number and
Number Systems*

1. Identify and generate equivalent forms of whole numbers; e.g., 36, $30 + 6$, 9×4 , $46 - 10$, number of inches in a yard.
2. Use place value concepts to represent whole numbers and decimals using numerals, words, expanded notation and physical models. For example:
 - a. Recognize 100 means “10 tens” as well as a single entity (1 hundred) through physical models and trading games.
 - b. Describe the multiplicative nature of the number system; e.g., the structure of 3205 as 3×1000 plus 2×100 plus 5×1 .
 - c. Model the size of 1000 in multiple ways; e.g., packaging 1000 objects into 10 boxes of 100, modeling a meter with centimeter and decimeter strips, or gathering 1000 pop-can tabs.
 - d. Explain the concept of tenths and hundredths using physical models, such as metric pieces, base ten blocks, decimal squares or money.
3. Use mathematical language and symbols to compare and order; e.g., less than, greater than, at most, at least, $<$, $>$, $=$, \leq , \geq .
4. Count money and make change using coins and paper bills to ten dollars.
5. Represent fractions and mixed numbers using words, numerals and physical models.
6. Compare and order commonly used fractions and mixed numbers using number lines, models (such as fraction circles or bars), points of reference (such as more or less than $\frac{1}{2}$), and equivalent forms found using physical or visual models.

*Meaning of
Operations*

7. Recognize and use decimal and fraction concepts and notations as related ways of representing parts of a whole or a set; e.g., 3 of 10 marbles are red can also be described as $\frac{3}{10}$ and 3 tenths are red.
8. Model, represent and explain multiplication; e.g., repeated addition, skip counting, rectangular arrays and area model. For example:
 - a. Use conventional mathematical symbols to write equations for word problems involving multiplication.
 - b. Understand that, unlike addition and subtraction, the factors in multiplication and division may have different units; e.g., 3 boxes of 5 cookies each.
9. Model, represent and explain division; e.g., sharing equally, repeated subtraction, rectangular arrays and area model. For example:
 - a. Translate contextual situations involving division into conventional mathematical symbols.
 - b. Explain how a remainder may impact an answer in a real-world situation; e.g., 14 cookies being shared by 4 children.
10. Explain and use relationships between operations, such as:

Grade Four

*Number and
Number Systems*

1. Identify and generate equivalent forms of fractions and decimals. For example:
 - a. Connect physical, verbal and symbolic representations of fractions, decimals and whole numbers; e.g., $\frac{1}{2}$, $\frac{5}{10}$, “five tenths,” 0.5, shaded rectangles with half, and five tenths.
 - b. Understand and explain that ten tenths is the same as one whole in both fraction and decimal form.
2. Use place value structure of the base-ten number system to read, write, represent and compare whole numbers through millions and decimals through thousandths.
3. Round whole numbers to a given place value.
4. Identify and represent factors and multiples of whole numbers through 100, and classify numbers as prime or composite.
5. Use models and points of reference to compare commonly used fractions.

*Meaning of
Operations*

6. Use associative and distributive properties to simplify and perform computations; e.g., use left to right multiplication and the distributive property to find an exact answer without paper and pencil, such as $5 \times 47 = 5 \times 40 + 5 \times 7 = 200 + 35 = 235$.
7. Recognize that division may be used to solve different types of problem situations and interpret the meaning of remainders; e.g., situations involving measurement, money.

*Computation
and Estimation*

8. Solve problems involving counting money and making change, using both coins and paper bills.
9. Estimate the results of computations involving whole numbers, fractions and decimals, using a variety of strategies.
10. Use physical models, visual representations, and paper and pencil to add and subtract decimals and commonly used fractions with like denominators.
11. Develop and explain strategies for performing computations mentally.
12. Analyze and solve multi-step problems involving addition, subtraction, multiplication and division using an organized approach, and verify and interpret results with respect to the original problem.
13. Use a variety of methods and appropriate tools for computing with whole numbers; e.g., mental math, paper and pencil, and calculator.
14. Demonstrate fluency in adding and subtracting whole numbers and in multiplying and dividing whole numbers by 1- and 2-digit numbers and multiples of ten.

Grade Five

Number and Number Systems

1. Use models and visual representation to develop the concept of ratio as part-to-part and part-to-whole, and the concept of percent as part-to-whole.
2. Use various forms of “one” to demonstrate the equivalence of fractions; e.g., $\frac{18}{24} = \frac{9}{12}$ x $\frac{2}{2} = \frac{3}{4}$ x $\frac{6}{6}$.
3. Identify and generate equivalent forms of fractions, decimals and percents.
4. Round decimals to a given place value and round fractions (including mixed numbers) to the nearest half.

Meaning of Operations

5. Recognize and identify perfect squares and their roots.
6. Represent and compare numbers less than 0 by extending the number line and using familiar applications; e.g., temperature, owing money.
7. Use commutative, associative, distributive, identity and inverse properties to simplify and perform computations.
8. Identify and use relationships between operations to solve problems.
9. Use order of operations, including use of parentheses, to simplify numerical expressions.

Computation and Estimation

10. Justify why fractions need common denominators to be added or subtracted.
11. Explain how place value is related to addition and subtraction of decimals; e.g., $0.2 + 0.14$; the two tenths is added to the one tenth because they are both tenths.
12. Use physical models, points of reference, and equivalent forms to add and subtract commonly used fractions with like and unlike denominators and decimals.
13. Estimate the results of computations involving whole numbers, fractions and decimals, using a variety of strategies.

Grade Six

*Number and
Number Systems*

1. Decompose and recompose whole numbers using factors and exponents (e.g., $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$), and explain why “squared” means “second power” and “cubed” means “third power.”
2. Find and use the prime factorization of composite numbers. For example:
 - a. Use the prime factorization to recognize the greatest common factor (GCF).
 - b. Use the prime factorization to recognize the least common multiple (LCM).
 - c. Apply the prime factorization to solve problems and explain solutions.
3. Explain why a number is referred to as being “rational,” and recognize that the expression $\frac{a}{b}$ can mean a parts of size $\frac{1}{b}$ each, a divided by b , or the ratio of a to b .
4. Describe what it means to find a specific percent of a number, using real-life examples.

*Meaning of
Operations*

5. Use models and pictures to relate concepts of ratio, proportion and percent, including percents less than 1 and greater than 100.
6. Use the order of operations, including the use of exponents, decimals and rational numbers, to simplify numerical expressions.
7. Use simple expressions involving integers to represent and solve problems; e.g., if a running back loses 15 yards on the first carry but gains 8 yards on the second carry, what is the net gain/loss?
8. Represent multiplication and division situations involving fractions and decimals with models and visual representations; e.g., show with pattern blocks what it means to take $2\frac{2}{3} \div \frac{1}{6}$.
9. Give examples of how ratios are used to represent comparisons; e.g., part-to-part, part-to-whole, whole-to-part.

*Computation
and Estimation*

10. Recognize that a quotient may be larger than the dividend when the divisor is a fraction; e.g., $6 \div \frac{1}{2} = 12$.
11. Perform fraction and decimal computations and justify their solutions; e.g., using manipulatives, diagrams, mathematical reasoning.
12. Develop and analyze algorithms for computing with fractions and decimals, and demonstrate fluency in their use.
13. Estimate reasonable solutions to problem situations involving fractions and decimals; e.g., $\frac{7}{8} + \frac{12}{13} \approx 2$ and $4.23 \times 5.8 \approx 25$.
14. Use proportional reasoning, ratios and percents to represent problem

Grade Seven

Number and Number Systems

1. Demonstrate an understanding of place value using powers of 10 and write large numbers in scientific notation.
2. Explain the meaning of exponents that are negative or 0.
3. Describe differences between rational and irrational numbers; e.g., use technology to show that some numbers (rational) can be expressed as terminating or repeating decimals and others (irrational) as non-terminating and non-repeating decimals.

Meaning of Operations

4. Use order of operations and properties to simplify numerical expressions involving integers, fractions and decimals.
5. Explain the meaning and effect of adding, subtracting, multiplying and dividing integers; e.g., how adding two integers can result in a lesser value.

Computation and Estimation

6. Simplify numerical expressions involving integers and use integers to solve real-life problems.
7. Solve problems using the appropriate form of a rational number (fraction, decimal or percent).
8. Develop and analyze algorithms for computing with percents and integers, and demonstrate fluency in their use.
9. Represent and solve problem situations that can be modeled by and solved using concepts of absolute value, exponents and square roots (for perfect squares).

Grade Eight

*Number and
Number Systems*

1. Use scientific notation to express large numbers and small numbers between 0 and 1.
2. Recognize that natural numbers, whole numbers, integers, rational numbers and irrational numbers are subsets of the real number system.

*Meaning of
Operations*

3. Apply order of operations to simplify expressions and perform computations involving integer exponents and radicals.
4. Explain and use the inverse and identity properties and use inverse relationships (addition/subtraction, multiplication/division, squaring/square roots) in problem solving situations.

*Computation
and Estimation*

5. Determine when an estimate is sufficient and when an exact answer is needed in problem situations, and evaluate estimates in relation to actual answers; e.g., very close, less than, greater than.
6. Estimate, compute and solve problems involving rational numbers, including ratio, proportion and percent, and judge the reasonableness of solutions.
7. Find the square root of perfect squares, and approximate the square root of non-perfect squares as consecutive integers between which the root lies; e.g., $\sqrt{130}$ is between 11 and 12.
8. Add, subtract, multiply, divide and compare numbers written in scientific notation.

Measurement Standard

Kindergarten

*Measurement
Units*

1. Identify units of time (day, week, month, year) and compare calendar elements; e.g., weeks are longer than days.

*Use
Measurement
Techniques and
Tools*

2. Compare and order objects of different lengths, areas, weights and capacities; and use relative terms, such as longer, shorter, bigger, smaller, heavier, lighter, more and less.
3. Measure length and volume (capacity) using uniform objects in the environment. For example, find:
 - a. how many paper clips long is a pencil;
 - b. how many small containers it takes to fill one big container using sand, rice, beans.
4. Order events based on time. For example:
 - a. activities that take a long or short time;
 - b. review what we do first, next, last;
 - c. recall what we did or plan to do yesterday, today, tomorrow.

Grade One

*Measurement
Units*

1. Recognize and explain the need for fixed units and tools for measuring length and weight; e.g., rulers and balance scales.
2. Tell time to the hour and half hour on digital and analog (dial) timepieces.
3. Order a sequence of events with respect to time; e.g., summer, fall, winter and spring; morning, afternoon and night.

*Use
Measurement
Techniques and
Tools*

4. Estimate and measure weight using non-standard units; e.g., blocks of uniform size.
5. Estimate and measure lengths using non-standard and standard units; i.e., centimeters, inches and feet.

Grade Two

*Measurement
Units*

1. Identify and select appropriate units of measure for:
 - a. length – centimeters, meters, inches, feet, or yards;
 - b. volume (capacity) – liters, cups, pints, or quarts;
 - c. weight – grams, ounces, or pounds;
 - d. time – hours, half-hours, quarter-hours, or minutes and time designations a.m. or p.m.
2. Establish personal or common referents for units of measure to make estimates and comparisons; e.g., the width of a finger is a centimeter, a large bottle of soda pop is 2 liters, a small paper clip weighs about one gram.
3. Describe and compare the relationships among units of measure, such as centimeters and meters; inches, feet and yards; cups, pints and quarts; ounces and pounds; and hours, half-hours, and quarter-hours; e.g., how many inches in a foot?

*Use
Measurement
Techniques and
Tools*

4. Tell time to the nearest minute interval on digital and to the nearest 5 minute interval on analog (dial) timepieces.
5. Estimate and measure the length and weight of common objects, using metric and U.S. customary units, accurate to the nearest unit.
6. Select and use appropriate measurement tools; e.g., a ruler to draw a segment 3 inches long, a measuring cup to place 2 cups of rice in a bowl, a scale to weigh 50 grams of candy.
7. Make and test predictions about measurements, using different units to measure the same length or volume.

Grade Three

*Measurement
Units*

1. Identify and select appropriate units for measuring:
 - a. length – miles, kilometers and other units of measure as appropriate.
 - b. volume (capacity) – gallons;
 - c. weight – ounces, pounds, grams, or kilograms;
 - d. temperature – degrees (Fahrenheit or Celsius).
2. Establish personal or common referents to include additional units; e.g., a gallon container of milk; a postage stamp is about a square inch.
3. Tell time to the nearest minute and find elapsed time using a calendar or a clock.
4. Read thermometers in both Fahrenheit and Celsius scales.

*Use
Measurement
Techniques and
Tools*

5. Estimate and measure length, weight and volume (capacity), using metric and U.S. customary units, accurate to the nearest $\frac{1}{2}$ or $\frac{1}{4}$ unit as appropriate.
6. Use appropriate measurement tools and techniques to construct a figure or approximate an amount of specified length, weight or volume (capacity); e.g., construct a rectangle with length $2\frac{1}{2}$ inches and width 3 inches, fill a measuring cup to the $\frac{3}{4}$ cup mark.
7. Make estimates for perimeter, area and volume using links, tiles, cubes and other models.

Grade Four

*Measurement
Units*

1. Relate the number of units to the size of the units used to measure an object; e.g., compare the number of cups to fill a pitcher to the number of quarts to fill the same pitcher.
2. Demonstrate and describe perimeter as surrounding and area as covering a two-dimensional shape, and volume as filling a three-dimensional object.
3. Identify and select appropriate units to measure:

*Use
Measurement
Techniques and
Tools*

- a. perimeter – string or links (inches or centimeters).
 - b. area – tiles (square inches or square centimeters).
 - c. Volume – cubes (cubic inches or cubic centimeters).
4. Develop and use strategies to find perimeter using string or links, area using tiles or a grid, and volume using cubes; e.g., count squares to find area of regular or irregular shapes on a grid, layer cubes in a box to find its volume.
 5. Make simple unit conversions within a measurement system; e.g., inches to feet, kilograms to grams, quarts to gallons.
 6. Write, solve and verify solutions to multi-step problems involving measurement.

Grade Five

*Measurement
Units*

1. Identify and select appropriate units to measure angles; i.e., degrees.
2. Identify paths between points on a grid or coordinate plane and compare the lengths of the paths; e.g., shortest path, paths of equal length.
3. Demonstrate and describe the differences between covering the faces (surface area) and filling the interior (volume) of three-dimensional objects.
4. Demonstrate understanding of the differences among linear units, square units and cubic units.

*Use
Measurement
Techniques and
Tools*

5. Make conversions within the same measurement system while performing computations.
6. Use strategies to develop formulas for determining perimeter and area of triangles, rectangles and parallelograms, and volume of rectangular prisms.
7. Use benchmark angles (e.g.; 45° , 90° , 120°) to estimate the measure of angles, and use a tool to measure and draw angles.

Grade Six

*Measurement
Units*

1. Understand and describe the difference between surface area and volume.

*Use
Measurement
Techniques and
Tools*

2. Use strategies to develop formulas for finding circumference and area of circles, and to determine the area of sectors; e.g., $\frac{1}{2}$ circle, $\frac{2}{3}$ circle, $\frac{1}{3}$ circle, $\frac{1}{4}$ circle.
3. Estimate perimeter or circumference and area for circles, triangles and quadrilaterals, and surface area and volume for prisms and cylinders by:
 - a. estimating lengths using string or links, areas using tiles or grid, and volumes using cubes;
 - b. measuring attributes (diameter, side lengths, or heights) and using established formulas for circles, triangles, rectangles, parallelograms and rectangular prisms.
4. Determine which measure (perimeter, area, surface area, volume) matches the context for a problem situation; e.g., perimeter is the context for fencing a garden, surface area is the context for painting a room.
5. Understand the difference between perimeter and area, and demonstrate that two shapes may have the same perimeter, but different areas or may have the same area, but different perimeters.
6. Describe what happens to the perimeter and area of a two-dimensional shape when the measurements of the shape are changed; e.g. length of sides are doubled.

Grade Seven

*Measurement
Units*

1. Select appropriate units for measuring derived measurements; e.g., miles per hour, revolutions per minute.
2. Convert units of area and volume within the same measurement system using proportional reasoning and a reference table when appropriate; e.g., square feet to square yards, cubic meters to cubic centimeters.

*Use
Measurement
Techniques and
Tools*

3. Estimate a measurement to a greater degree of precision than the tool provides.
4. Solve problems involving proportional relationships and scale factors; e.g., scale models that require unit conversions within the same measurement system.
5. Analyze problem situations involving measurement concepts, select appropriate strategies, and use an organized approach to solve narrative and increasingly complex problems.
6. Use strategies to develop formulas for finding area of trapezoids and volume of cylinders and prisms.
7. Develop strategies to find the area of composite shapes using the areas of triangles, parallelograms, circles and sectors.
8. Understand the difference between surface area and volume and demonstrate that two objects may have the same surface area, but different volumes or may have the same volume, but different surface areas.
9. Describe what happens to the surface area and volume of a three-dimensional object when the measurements of the object are changed; e.g., length of sides are doubled.

Grade Eight

*Measurement
Units*

1. Compare and order the relative size of common U.S. customary units and metric units; e.g., mile and kilometer, gallon and liter, pound and kilogram.
2. Use proportional relationships and formulas to convert units from one measurement system to another; e.g., degrees Fahrenheit to degrees Celsius.

*Use
Measurement
Techniques and
Tools*

3. Use appropriate levels of precision when calculating with measurements.
4. Derive formulas for surface area and volume and justify them using geometric models and common materials. For example, find:
 - a. the surface area of a cylinder as a function of its height and radius;
 - b. that the volume of a pyramid (or cone) is one-third of the volume of a prism (or cylinder) with the same base area and height.
5. Determine surface area for pyramids by analyzing their parts.
6. Solve and determine the reasonableness of the results for problems involving rates and derived measurements, such as velocity and density, using formulas, models and graphs.
7. Apply proportional reasoning to solve problems involving indirect measurements or rates.
8. Find the sum of the interior and exterior angles of regular convex polygons with and without measuring the angles with a protractor.
9. Demonstrate understanding of the concepts of perimeter, circumference and area by using established formula for triangles, quadrilaterals, and circles to determine the surface area and volume of prisms, pyramids, cylinders, spheres and cones. (Note: Only volume should be calculated for spheres and cones.)
10. Use conventional formulas to find the surface area and volume of prisms, pyramids and cylinders and the volume of spheres and cones to a specified level of precision.

Geometry and Spatial Sense Standard

Kindergarten

*Characteristics
and Properties*

1. Identify and sort two- dimensional shapes and three- dimensional objects.
For example:
 - a. Identify and describe two-dimensional figures and three-dimensional objects from the environment using the child’s own vocabulary.
 - b. Sort shapes and objects into groups based on student-defined categories.
 - c. Select all shapes or objects of one type from a group.
 - d. Build two-dimensional figures using paper shapes or tangrams; build simple three-dimensional objects using blocks.

*Spatial
Relationships*

2. Name and demonstrate the relative position of objects as follows:
 - a. place objects over, under, inside, outside, on, beside, between, above, below, on top of, upside-down, behind, in back of, in front of;
 - b. describe placement of objects with terms such as on, inside, outside, above, below, over, under, beside, between, in front of, behind.

Grade One

*Characteristics
and Properties*

1. Identify, compare, and sort two-dimensional shapes; i.e., square, circle, ellipse, triangle, rectangle, rhombus, trapezoid, parallelogram, pentagon, and hexagon. For example:
 - a. Recognize and identify triangles and rhombuses independent of position, shape or size;
 - b. Describe two-dimensional shapes using attributes such as number of sides and number of vertices (corners, or angles).
2. Create new shapes by combining or cutting apart existing shapes.
3. Identify the shapes of the faces of three-dimensional objects.

*Spatial
Relationships*

4. Extend the use of location words to include distance (near, far, close to) and directional words (left, right).
5. Copy figures and draw simple two-dimensional shapes from memory.

Grade Two

*Characteristics
and Properties*

1. Identify, describe, compare, and sort three-dimensional objects (i.e., cubes, spheres, prisms, cones, cylinders and pyramids) according to the shape of the faces or the numbers of faces, edges, or vertices.
2. Predict what new shapes will be formed by combining or cutting apart existing shapes.
3. Recognize two-dimensional shapes and three-dimensional objects from different positions.

*Spatial
Relationships*

4. Identify and determine whether two-dimensional shapes are congruent (same shape and size) or similar (same shape different size) by copying or using superposition (lay one thing on top of another).

*Transformations
and Symmetry*

5. Create and identify two-dimensional figures with line symmetry; e.g., what letter shapes, logos, polygons are symmetrical?

Grade Three

*Characteristics
and Properties*

1. Analyze and describe properties of two-dimensional shapes and three-dimensional objects using terms such as vertex, edge, angle, side and face.
2. Identify and describe the relative size of angles with respect to right angles as follows:

*Spatial
Relationships*

*Transformations
and Symmetry*

*Visualization
and Geometric
Models*

- a. Use physical models, like straws, to make different sized angles by opening and closing the sides, not by changing the side lengths.
 - b. Identify, classify and draw right, acute, obtuse and straight angles.
3. Find and name locations on a labeled grid or coordinate system; e.g., a map or graph.
 4. Draw lines of symmetry to verify symmetrical two-dimensional shapes.
 5. Build a three-dimensional model of an object composed of cubes; e.g., construct a model based on an illustration or actual object.

Grade Four

*Characteristics
and Properties*

1. Identify, describe and model intersecting, parallel and perpendicular lines and line segments; e.g., use straws or other material to model lines.
2. Describe, classify, compare and model two- and three-dimensional objects using their attributes.
3. Identify similarities and differences of quadrilaterals; e.g., squares, rectangles, parallelograms and trapezoids.
4. Identify and define triangles based on angle measures (equiangular, right, acute and obtuse triangles) and side lengths (isosceles, equilateral and scalene triangles).

*Spatial
Relationships*

5. Describe points, lines and planes, and identify models in the environment.
6. Specify locations and plot ordered pairs on a coordinate plane, using first quadrant points.

*Transformations
and Symmetry*

7. Identify, describe and use reflections (flips), rotations (turns), and translations (slides) in solving geometric problems; e.g., use transformations to determine if 2 shapes are congruent.

*Visualization
and Geometric
Models*

8. Use geometric models to solve problems in other areas of mathematics, such as number (multiplication/division) and measurement (area, perimeter, border).

Grade Five

*Characteristics
and Properties*

1. Draw circles, and identify and determine relationships among the radius, diameter, center and circumference; e.g., radius is half the diameter, the ratio of the circumference of a circle to its diameter is an approximation of π .
2. Use standard language to describe line, segment, ray, angle, skew, parallel and perpendicular.
3. Label vertex, rays, interior and exterior for an angle.
4. Describe and use properties of congruent figures to solve problems.
5. Use physical models to determine the sum of the interior angles of triangles and quadrilaterals.

*Spatial
Relationships*

*Visualization
and Geometric
Models*

6. Extend understanding of coordinate system to include points whose x or y values may be negative numbers.
7. Understand that the measure of an angle is determined by the degree of rotation of an angle side rather than the length of either side.
8. Predict what three-dimensional object will result from folding a two-dimensional net, then confirm the prediction by folding the net.

Grade Six

Characteristics and Properties

1. Classify and describe two-dimensional and three-dimensional geometric figures and objects by using their properties; e.g., interior angle measures, perpendicular/parallel sides, congruent angles/sides.
2. Use standard language to define geometric vocabulary: vertex, face, altitude, diagonal, isosceles, equilateral, acute, obtuse, etc.
3. Use multiple classification criteria to classify triangles; e.g., right scalene triangle.
4. Identify and define relationships between planes; i.e., parallel, perpendicular and intersecting.

Spatial Relationships

5. Predict and describe sizes, positions and orientations of two-dimensional shapes after transformations such as reflections, rotations, translations and dilations.

Transformation and Symmetry

6. Draw similar figures that model proportional relationships; e.g., model similar figures with a 1 to 2 relationship by sketching two of the same figure, one with corresponding sides twice the length of the other.

Visualization and Geometric Models

7. Build three-dimensional objects built with cubes and sketch the two-dimensional representations of each side; i.e., projection sets.

Grade Seven

*Characteristics
and Properties*

1. Use proportional reasoning to describe and express relationships between parts and attributes of similar and congruent figures.
2. Determine sufficient (not necessarily minimal) properties that define a specific two-dimensional figure or three-dimensional object. For example:
 - a. Determine when one set of figures is a subset of another; e.g., all squares are rectangles.
 - b. Develop a set of properties that eliminates all but the desired figure; e.g., only squares are quadrilaterals with all sides congruent and all angles congruent.
3. Use and demonstrate understanding of the properties of triangles. For example:
 - a. Use Pythagorean Theorem to solve problems involving right triangles.
 - b. Use triangle angle sum relationships to solve problems.

*Spatial
Relationships*

4. Determine necessary conditions for congruence of triangles.

*Transformation
and Symmetry*

5. Apply properties of congruent or similar triangles to solve problems involving missing lengths and angle measures.
6. Determine and use scale factors for similar figures to solve problems using proportional reasoning.

*Visualization
and Geometric
Models*

7. Identify the line and rotation symmetries of two-dimensional figures to solve problems.
8. Perform translations, reflections, rotations and dilations of two-dimensional figures using a variety of methods (paper folding, tracing, graph paper).
9. Draw representations of three-dimensional geometric objects from different views.

Grade Eight

*Characteristics
and Properties*

1. Make and test conjectures about characteristics and properties (e.g., sides, angles, symmetry) of two-dimensional figures and three-dimensional objects.
2. Recognize the angles formed and the relationship between the angles when two lines intersect and when parallel lines are cut by a transversal.
3. Use proportions in several forms to solve problems involving similar figures (part-to-part, part-to-whole, corresponding sides between figures).

*Spatial
Relationships*

4. Represent and analyze shapes using coordinate geometry; e.g., given three vertices and the type of quadrilateral, find the coordinates of the fourth vertex.

*Transformation
and Symmetry*

5. Draw the results of translations, reflections, rotations and dilations of objects in the coordinate plane, and determine properties that remain fixed; e.g., lengths of sides remain the same under translations.

*Visualization
and Geometric
Models*

6. Draw nets for a variety of prisms, pyramids, cylinders and cones.

Patterns, Functions and Algebra Standard

Kindergarten

*Use Patterns,
Relations and
Functions*

1. Sort, classify and order objects by size, number and other properties. For example:
 - a. Identify how objects are alike and different.
 - b. Order three events or objects according to a given attribute, such as time or size.
 - c. Recognize and explain how objects can be classified in more than one way.
 - d. Identify what attribute was used to sort groups of objects that have already been sorted.
2. Identify, create, extend and copy sequences of sounds (such as musical notes), shapes (such as buttons, leaves or blocks), motions (such as hops or skips), and numbers from 1 to 10.

*Use Algebraic
Representations*

3. Describe orally the pattern of a given sequence.
4. Model a problem situation using physical materials.

Grade One

*Use Patterns,
Relations and
Functions*

1. Sort, classify and order objects by two or more attributes, such as color and shape, and explain how objects were sorted.
2. Extend sequences of sounds, shapes or simple number patterns, and create and record similar patterns. For example:
 - a. Analyze and describe patterns with multiple attributes using numbers and shapes; e.g., AA, B, aa, b, AA, B, aa, b,...
 - b. Continue repeating and growing patterns with materials, pictures and geometric items; e.g., XO, XOO, XOOO, XOOOO.
3. Describe orally the basic unit or general plan of a repeating or growing pattern.

*Use Algebraic
Representations*

4. Solve open sentences by representing an expression in more than one way using the commutative property; e.g., $4 + 5 = 5 + 4$ or the number of blue balls plus red balls is the same as the number of red balls plus blue balls ($R+B=B+R$).
5. Describe orally and model a problem situation using words, objects or number phrase or sentence.

Grade Two

*Use Patterns,
Relations and
Functions*

1. Extend simple number patterns (both repeating and growing patterns), and create similar patterns using different objects, such as using physical materials or shapes to represent numerical patterns.
2. Use patterns to make generalizations and predictions; e.g., determine a missing element in a pattern.
3. Create new patterns with consistent rules or plans, and describe the rule or general plan of existing patterns.

*Use Algebraic
Representations*

4. Use objects, pictures, numbers and other symbols to represent a problem situation.
5. Understand equivalence and extend the concept to situations involving symbols; e.g., $4 + 5 = 9$ and $9 = 4 + 5$ and $4 + 5 = 3 + 6 = \Delta + \square \dots$
6. Use symbols to represent unknown quantities and identify values for symbols in an expression or equation using addition and subtraction; e.g., $\square + \bigcirc = 10$, $\Delta - 2 = 4$.

Analyze Change

7. Describe qualitative and quantitative changes, especially those involving addition and subtraction; e.g., a student growing taller versus a student growing two inches in one year.

Grade Three

*Use Patterns,
Relations and
Functions*

1. Extend multiplicative and growing patterns, and describe the pattern or rule in words.
2. Analyze and replicate arithmetic sequences with and without a calculator.
3. Use patterns to make predictions, identify relationships, and solve problems.

*Use Algebraic
Representations*

4. Model problem situations using objects, pictures, tables, numbers, letters and other symbols.
5. Write, solve and explain simple mathematical statements, such as $7 + \square > 8$ or $\Delta + 8 = 10$.
6. Express mathematical relationships as equations and inequalities.

Analyze Change

7. Create tables to record, organize and analyze data to discover patterns and rules.
8. Identify and describe quantitative changes, especially those involving addition and subtraction; e.g., the height of water in a glass becoming 1 centimeter lower each week due to evaporation.

Grade Four

*Use Patterns,
Relations and
Functions*

1. Use models and words to describe, extend and make generalizations of patterns and relationships occurring in computation, numerical patterns, geometry, graphs and other applications.
2. Represent and analyze patterns and functions using words, tables and graphs.

*Use Algebraic
Representation*

3. Construct a table of values to solve problems associated with a mathematical relationship.
4. Use rules and variables to describe patterns and other relationships.

Analyze Change

5. Represent mathematical relationships with equations or inequalities.
6. Describe how a change in one variable affects the value of a related variable; e.g., as one increases the other increases or as one increases the other decreases.

Grade Five

*Use Patterns,
Relations and
Functions*

1. Justify a general rule for a pattern or a function by using physical materials, visual representations, words, tables or graphs.
2. Use calculators or computers to develop patterns, and generalize them using tables and graphs.

*Use Algebraic
Representations*

3. Use variables as unknown quantities in general rules when describing patterns and other relationships.
4. Create and interpret the meaning of equations and inequalities representing problem situations.
5. Model problems with physical materials and visual representations, and use models, graphs and tables to draw conclusions and make predictions.

Analyze Change

6. Describe how the quantitative change in a variable affects the value of a related variable; e.g., describe how the rate of growth varies over time, based upon data in a table or graph.

Grade Six

*Use Patterns,
Relations and
Functions*

1. Represent and analyze patterns, rules and functions, using physical materials, tables and graphs.
2. Use words and symbols to describe numerical and geometric patterns, rules and functions.

*Use Algebraic
Representations*

3. Recognize and generate equivalent forms of algebraic expressions, and explain how the commutative, associative and distributive properties can be used to generate equivalent forms; e.g., perimeter as $2(l + w)$ or $2l + 2w$.
4. Solve simple linear equations and inequalities using physical models, paper and pencil, tables and graphs.
5. Produce and interpret graphs that represent the relationship between two variables.
6. Evaluate simple expressions by replacing variables with given values, and use formulas in problem-solving situations.

Analyze Change

7. Identify and describe situations with constant or varying rates of change, and compare them.
8. Use technology to analyze change; e.g., use computer applications or graphing calculators to display and interpret rate of change.

Grade Seven

*Use Patterns,
Relations and
Functions*

1. Represent and analyze patterns, rules and functions with words, tables, graphs and simple variable expressions.
2. Generalize patterns by describing in words how to find the next term.
3. Recognize and explain when numerical patterns are linear or nonlinear progressions; e.g., 1,3,5,7... is linear and 1,3,4,8,16... is nonlinear.

*Use Algebraic
Representations*

4. Create visual representations of equation-solving processes that model the use of inverse operations.
5. Represent linear equations by plotting points in the coordinate plane.
6. Represent inequalities on a number line or a coordinate plane.
7. Justify that two forms of an algebraic expression are equivalent, and recognize when an expression is simplified; e.g., $4m = m + m + m + m$
or
 $a \cdot 5 + 4 = 5a + 4$.
8. Use formulas in problem-solving situations.

Analyze Change

9. Recognize a variety of uses for variables; e.g., placeholder for an unknown quantity in an equation, generalization for a pattern, formula.
10. Analyze linear and simple nonlinear relationships to explain how a change in one variable results in the change of another.
11. Use graphing calculators or computers to analyze change; e.g., distance-time relationships.

Grade Eight

*Use Patterns,
Relations and
Functions*

1. Relate the various representations of a relationship; i.e., relate a table to graph, description and symbolic form.
2. Generalize patterns and sequences by describing how to find the n th term.
3. Identify functions as linear or nonlinear based on information given in a table, graph or equation.

*Use Algebraic
Representations*

4. Extend the uses of variables to include covariants where y depends on x .
5. Use physical models to add and subtract monomials and polynomials, and to multiply a polynomial by a monomial.
6. Describe the relationship between the graph of a line and its equation, including being able to explain the meaning of slope as a constant rate of change and y -intercept in real-world problems.
7. Use symbolic algebra (equations and inequalities), graphs and tables to represent situations and solve problems.
8. Write, simplify and evaluate algebraic expressions (including formulas) to generalize situations and solve problems.
9. Solve linear equations and inequalities graphically, symbolically and using technology.
10. Solve 2 by 2 systems of linear equations graphically and by simple substitution.
11. Interpret the meaning of the solution of a 2 by 2 system of equations; i.e., point, line, no solution.
12. Solve simple quadratic equations graphically; e.g., $y = x^2 - 16$.
13. Compute and interpret slope, midpoint and distance given a set of ordered pairs.

Analyze Change

14. Differentiate and explain types of changes in mathematical relationships, such as linear vs. nonlinear, continuous vs. noncontinuous, direct variation vs. inverse variation.
15. Describe and compare how changes in an equation affects the related graphs; e.g., for a linear equation changing the coefficient of x affects the slope and changing the constant affects the intercepts.
16. Use graphing calculators or computers to analyze change; e.g., interest compounded over time as a nonlinear growth pattern.

Data Analysis and Probability

Kindergarten

*Data
Collection*

Gather and sort data in response to questions posed by teacher and students; e.g., how many sisters and brothers, what color shoes.

Arrange objects in a floor or table graph according to attributes, such as use, size, color, or shape.

*Statistical
Methods*

Select the category or categories that have the most or fewest objects in a floor or table graph.

Grade One

Data Collection

1. Identify multiple categories for sorting data.
2. Collect and organize data into charts using tally marks.
3. Display data in picture graphs with units of 1 and bar graphs with intervals of 1.
4. Read and interpret charts, picture graphs and bar graphs as sources of information to identify main ideas, draw conclusions, and make predictions.

*Statistical
Methods*

5. Construct a question that can be answered by using information from a graph.
6. Arrange five objects by an attribute, such as size or weight, and identify the ordinal position of each object.

Probability

7. Answer questions about the number of objects represented in a picture graph, bar graph or table graph; e.g., category with most, how many more in a category compared to another, how many altogether in two categories.
8. Describe the likelihood of simple events as possible/impossible and more likely/less likely; e.g., when using spinners or number cubes in classroom activities.

Grade Two

- Data Collection*
1. Pose questions, use observations, interviews and surveys to collect data, and organize data in charts, picture graphs and bar graphs.
 2. Read, interpret and make comparisons and predictions from data represented in charts, line plots, picture graphs and bar graphs.
 3. Read and construct simple timelines to sequence events.
- Statistical Methods*
4. Write a few sentences to describe and compare categories of data represented in a chart or graph, and make statements about the data as a whole.
 5. Identify untrue or inappropriate statements about a given set of data.
 6. Recognize that data may vary from one population to another; e.g., favorite TV shows of students and of parents.
- Probability*
7. List some of the possible outcomes of a simple experiment, and predict whether given outcomes are more, less or equally likely to occur.
 8. Use physical models and pictures to represent possible arrangements of 2 or 3 objects.

Grade Three

- Data Collection*
1. Collect and organize data from an experiment, such as recording and classifying observations or measurements, in response to a question posed.
 2. Draw and interpret picture graphs in which a symbol or picture represents more than one object.
 3. Read, interpret and construct bar graphs with intervals greater than one.
 4. Support a conclusion or prediction orally and in writing, using information in a table or graph.
 5. Match a set of data with a graphical representation of the data.
 6. Translate information freely among charts, tables, line plots, picture graphs and bar graphs; e.g., create a bar graph from the information in a chart.
- Statistical Methods*
7. Analyze and interpret information represented on a timeline.
 8. Identify the mode of a data set and describe the information it gives about a data set.
- Probability*
9. Conduct a simple experiment or simulation of a simple event, record the results in a chart, table or graph, and use the results to draw conclusions about the likelihood of possible outcomes.
 10. Use physical models, pictures, diagrams and lists to solve problems involving possible arrangements or combinations of two to four objects.

Grade Four

- Data Collection*
1. Create a plan for collecting data for a specific purpose.
 2. Represent and interpret data using tables, bar graphs, line plots and line graphs.
 3. Interpret and construct Venn diagrams to sort and describe data.
 4. Compare different representations of the same data to evaluate how well each representation shows important aspects of the data, and identify appropriate ways to display the data.
 5. Propose and explain interpretations and predictions based on data displayed in tables, charts and graphs.
- Statistical Methods*
6. Describe the characteristics of a set of data based on a graphical representation, such as range of the data, clumps of data, and holes in the data.
 7. Identify the median of a set of data and describe what it indicates about the data.
 8. Use range, median and mode to make comparisons among related sets of data.
- Probability*
9. Conduct simple probability experiments and draw conclusions from the results; e.g., rolling number cubes or drawing marbles from a bag.
 10. Represent the likelihood of possible outcomes for chance situations; e.g., probability of selecting a red marble from a bag containing 3 red and 5 white marbles.
 11. Relate the concepts of impossible and certain-to-happen events to the numerical values of 0 (impossible) and 1 (certain).
 12. Place events in order of likelihood and use a diagram or appropriate language to compare the chance of each event occurring; e.g. impossible, unlikely, equal, likely, certain.

- List and count all possible combinations using one member from each of several sets, each containing 2 or 3 members; e.g., the number of possible outfits from 3 shirts, 2 shorts and 2 pair of shoes.

Grade Five

- Data Collection*
- Read, construct and interpret frequency tables, circle graphs and line graphs.
 - Select and use a graph that is appropriate for the type of data to be displayed; e.g., numerical vs. categorical data, discrete vs. continuous data.
 - Read and interpret increasingly complex displays of data, such as double bar graphs.
 - Determine appropriate data to be collected to answer questions posed by students or teacher, collect and display data, and clearly communicate findings.
- Statistical Methods*
- Modify initial conclusions, propose and justify new interpretations and predictions as additional data are collected.
- Probability*
- Determine and use the range, mean, median and mode, and explain what each does and does not indicate about the set of data.
 - List and explain all possible outcomes in a given situation.
 - Identify the probability of events within a simple experiment, such as three chances out of eight.
 - Use 0,1 and ratios between 0 and 1 to represent the probability of outcomes for an event, and associate the ratio with the likelihood of the outcome.
 - Compare what should happen (theoretical/expected results) with what did happen (experimental/actual results) in a simple experiment.
 - Make predictions based on experimental and theoretical probabilities.

Grade Six

- Data Collection*
1. Read, construct and interpret line graphs, circle graphs and histograms.
 2. Select, create and use graphical representations that are appropriate for the type of data collected.
 3. Compare representations of the same data in different types of graphs, such as a bar graph and circle graph.
- Statistical Methods*
4. Understand the different information provided by measures of center (mean, mode and median) and measures of spread (range).
 5. Describe the frequency distribution of a set of data, as shown in a histogram or frequency table, by general appearance or shape; e.g., number of modes, middle of data and level of symmetry, outliers.
 6. Make logical inferences from statistical data.
- Probability*
7. Design an experiment to test a theoretical probability and explain how the results may vary.

Grade Seven

- Data Collection*
1. Read, create and interpret box-and-whisker plots, stem-and-leaf plots, and other types of graphs, when appropriate.
 2. Analyze how decisions about graphing affect the graphical representation; e.g., scale, size of classes in a histogram, number of categories in a circle graph.
- Statistical Methods*
3. Analyze a set of data by using and comparing combinations of measures of center (mean, mode, median) and measures of spread (range, quartile, interquartile range), and describe how the inclusion or exclusion of outliers affects those measures.
 4. Construct opposing arguments based on analysis of the same data, using different graphical representations.
 5. Compare data from two or more samples to determine how sample selection can influence results.
 6. Identify misuses of statistical data in articles, advertisements, and other media.
- Probability*
7. Compute probabilities of compound events; e.g., multiple coin tosses or multiple rolls of number cubes, using such methods as organized lists, tree diagrams and area models.
 8. Make predictions based on theoretical probabilities, design and conduct an experiment to test the predictions, compare actual results to predicted results, and explain differences.

Grade Eight

- Data Collection*
1. Use, create and interpret scatterplots and other types of graphs as appropriate.
 2. Evaluate different graphical representations of the same data to determine which is the most appropriate representation for an identified purpose; e.g., line graph for change over time, circle graph for part-to-whole comparison, scatterplot for relationship between two variants.
 3. Differentiate between discrete and continuous data and appropriate ways to represent each.
- Statistical Methods*
4. Compare two sets of data using measures of center (mean, mode, median) and measures of spread (range, quartiles, interquartile range, percentiles).
 5. Explain the mean's sensitivity to extremes and its use in comparison with the median and mode.
 6. Make conjectures about possible relationship in a scatterplot and approximate line of best fit.
 7. Identify different ways of selecting samples, such as survey response, random sample, representative sample and convenience sample.
 8. Describe how the relative size of a sample compared to the target population affects the validity of predictions.
- Probability*
9. Construct convincing arguments based on analysis of data and interpretation of graphs.
 10. Calculate the number of possible outcomes for a situation, recognizing and accounting for when items may occur more than once or when order is important.
 11. Demonstrate an understanding that the probability of either of two disjoint events occurring can be found by adding the probabilities for each and that the probability of one independent event following another can be found by multiplying the probabilities.

*Note: The benchmarks for
mathematical processes*